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# On The Non-homogeneous Ternary Bi-quadratic Equation

$$xz(x-z)=y^4$$

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Abstract:

This paper focuses on finding non-zero distinct integer solutions to the non-homogeneous ternary bi-quadratic diophantine equation given by $xz(x-z)=y^4$. Different sets of integer solutions are presented.

Key words: non-homogeneous bi-quadratic, ternary bi-quadratic ,integer solutions

Notations:

$$t_{m,n}=n(1+\frac{(n-1)(m-2)}{2}), P_n^3=\frac{n(n+1)(n+2)}{6}, P_n^5=\frac{n^2(n+1)}{2}$$

$$P_n^4=\frac{n(n+1)(2n+1)}{6}, CP_n^4=\frac{(2n^3+n)}{3}, CP_n^{12}=2n^3-n$$

Introduction:

The Diophantine equations are rich in variety and offer an unlimited field for research [1-4]. In particular refer [5-28] for a few problems on Biquadratic equation with 3 unknowns. This paper concerns with yet another interesting Biquadratic Diophantine equation with three variables given by $xz(x-z)=y^4$ for determining its infinitely many non-zero distinct integral solutions.

Method of analysis:

The non-homogeneous ternary bi-quadratic equation under consideration is

$$xz(x-z)=y^4 \quad (1)$$

Introduction of the linear transformations

$$x = u + v, z = u - v, y = 2v, u \neq v \neq 0 \qquad (2)$$

in (1) leads to

$$u^2 = v^2(1+8v)$$

which is satisfied by

$$v = \frac{n(n+1)}{2}, u = \frac{n(n+1)(2n+1)}{2}$$

In view of (2), the corresponding integer solutions to (1) are given by

$$x = n(n+1)^2, z = n^2(n+1, y = n(n+1) \qquad (3)$$

A few interesting relations among the solutions are exhibited below:

(i). $x + z = 6P_n^4$

(ii). $x + y + z = 12P_n^3 - 4t_{3,n}$

(iii). $x + z - 3CP_n^4$ is a square multiple of 3

(iv). $x + z - CP_n^{12} \equiv 2(\bmod 3)$

(v). $x + z - CP_n^{12} - t_{8,n} \equiv 0(\bmod 4)$

(Vi).Each of the following is a perfect square:

$$\frac{yx}{z}, \frac{zx}{y}, \frac{zy}{x}$$

(Vii). $x - y = 2P_n^5$

(Viii). $z - y = 6P_{n-1}^3$

(IX). $x^2 - z^2 = 12P_n^4 * t_{3,n}$

(X). $x + y + z - 3CP_n^4 - t_{10,n} \equiv 0(\bmod 4)$

It is worth mentioning that, apart from (2), one may employ other choices of linear transformations leading to different sets of solutions . For simplicity and brevity , we present below a few sets of solutions to (1) :

Set 1:

$$x = (k+8)(k^2-64), z = (k-8)(k^2-64), y = 2(k^2-64)$$

Set 2:

$$x = (5k-1)(75k^2-75k+12), z = (5k-4)(75k^2-75k+12), y = (75k^2-75k+12)$$

Set 3:

$$x = (k-128)(k+128)^2, z = (k+128)(k-128)^2, y = 4(k^2-128^2)$$

Set 4:

$$x = (3k+3)(45k^2+15k-30), z = (3k-2)(45k^2+15k-30), y = (45k^2+15k-30)$$

Set 5:

$$x = (45k^2+45k-20)(3k+4), z = (45k^2+45k-20)(3k-1), y = (45k^2+45k-20)$$

Conclusion:

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous bi-quadratic diophantine equation with three unknowns given by $xz(x-z) = y^4$ .One may search for other sets of integer solutions to the considered equation as well as other choices of the fourth degree diophantine equations with multi-variables

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